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Question Paper Code : 53315

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Sixth/Seventh Semester

Mechanical Engineering

ME 6603 — FINITE ELEMENT ANALYSIS

(Common to B.E. Mechanical Engineering (Sandwich)/Automobile Engineering/Manufacturing Engineering/Mechanical and Automation Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The general Weighted Residual Technique is expressed as $\int_{\Omega} R(x) w_i(x) dx = 0$ $i = 1, 2, ..., n$. Identify the weighting function associated with each of the WRMs.

2. Distinguish between Essential and Natural Boundary conditions with suitable examples.

3. What are the differences between a beam element and a bar element?

4. Derive the shape functions for a 1D three noded element.

5. Distinguish between CST and LST elements.

6. Write the stiffness matrix used for the torsion problem of a square shaft assuming three noded triangular elements of area A.

7. What is meant by Constitutive Matrix? Write the same for Plane Stress Analysis.

8. Brief the type of element that is best suited for analyzing a thin dome shaped structure subjected to out of plane load.

9. Sketch and write the advantages of Serendipity elements.

10. What is the significance of Jacobian of transformation?

PART B — (5 × 13 = 65 marks)

11. (a) A 50 mm long Aluminium pin fin of diameter 1 mm is attached to a wall that is maintained at 300°C. It is subjected to both conduction and convection heat transfer. The thermal conductivity k of Aluminium is 200 W/m°C and the convective heat transfer coefficient h is 20 W/m²°C. The free end of the fin is insulated. Determine using any Weighted Residual technique or the Ritz technique the temperature distribution along the fin and hence the temperature at the tip. The Governing differential equation for the fin is given by

$$\frac{d}{dx}\left(-kA\frac{dT}{dx}\right)+hp(T-T_{\infty})=0$$

Boundary Conditions:

(i) $T(0)=300°C$

(ii) $\left(\frac{dT}{dx}\right)_{x=50}=0$.

Or

(b) Determine the variation of displacement along a bar of varying cross section of length 90 cm. The bar is attached to a wall and suspended vertically. It carries a load of 20 kN at the tip. E = 210 GPa, γ = 0.0785 N/cm³. The bar is of rectangular cross section of side 5 cm × 3 cm at the fixed end and 3 cm × 3 cm at the free end. The displacement at the tip of the bar due to the point load and its own self weight is to be determined.

(i) How will you mathematically model this problem? (2)

(ii) What is the difference between the use of weighted residual technique, Ritz technique and the finite element technique for solving the above problem? (2)

(iii) Take at least 2 elements of equal length and solve for the displacement. What is the displacement at the tip of the bar? (9)

12. (a) Determine the maximum deflection and slope for the simply supported beam subjected to uniformly supported load 'q' as shown in Fig. 12 (a).

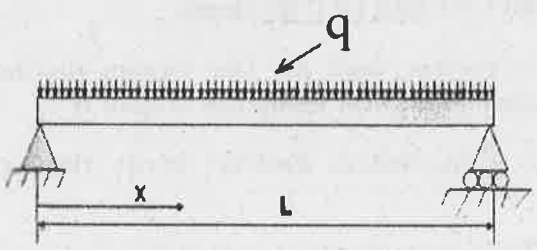

Fig. 12 (a)

Or

(b) Determine the first two natural frequencies of longitudinal vibration of the stepped steel bar shown in Fig. 12 (b). Use the mesh shown. All dimensions are in mm. $E = 200$ GPa and $\rho = 0.78$ kg/cc.

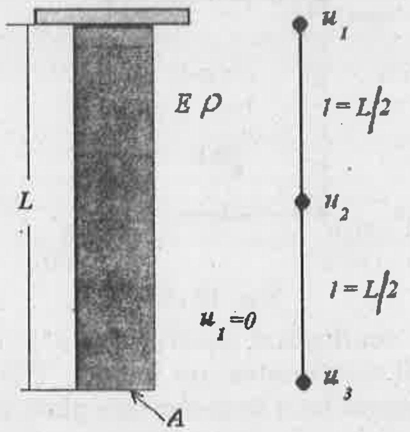

Fig. 12 (b)

13. (a) A member of rectangular cross section 1 cm × 0.5 cm, as shown in Fig. 13 (a), is to be analysed for determining the stress distribution. Considering geometric and boundary condition symmetry, 1/4th of the cross section was modeled using equisized triangular elements. The element matrices for a triangle whose nodal coordinates are (0,0), (0.25,0) and (0.25, 0.25) are given below. Explain why 1/4th of the cross section has to be considered and give the finite element mesh if the stiffness matrix given below is to be used. Carry out the assembly and solve for the unknown stress function values and explain how the shear stress distribution is to be determined. Where will the stress be the highest?

$$[K] = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \{r\} = \begin{Bmatrix} 29.1 \\ 29.1 \\ 29.1 \end{Bmatrix}$$

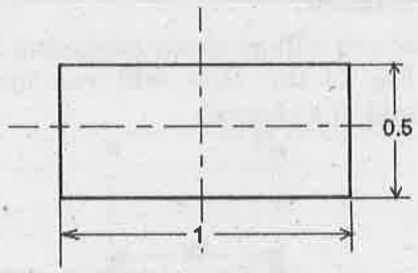

Fig. 13 (a)

Or

(b) (i) Determine the temperature at the location (2, 1) in a square plate with the data shown in Fig. 13 (b). Draw the 125°C isotherm using at least three points. (8)

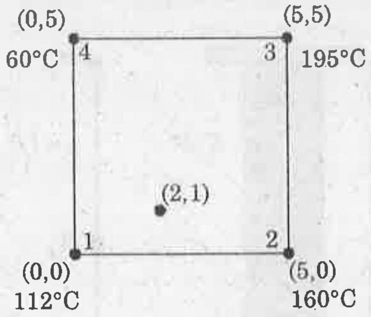

Fig. 13 (b)

(ii) Derive the conduction matrix for a 3 noded triangular element whose nodal coordinates are known. The element is to be used for two dimensional heat transfer in a plate fin. (5)

14. (a) It is required to determine the transverse displacement and the stresses induced in the plate shown in Fig. 14 (a) using a one-element idealization. Determine the constitutive matrix and the strain displacement matrix and hence the stiffness matrix and the load vector. Assume $E = 205$ GPa, $\mu = 0.33$, and $t = 10$ mm.

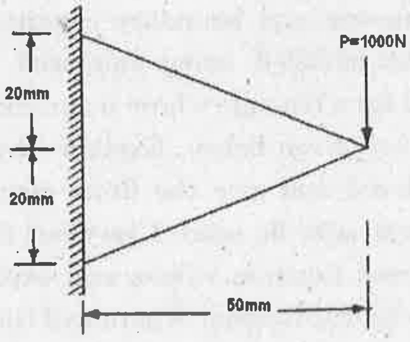

Fig. 14 (a)

Or

(b) (i) Give the Strain displacement relations for axisymmetric analysis and hence derive the Strain displacement matrix for a linear triangular element. (7)

(ii) Explain how you will go about analysing the Belleville (Disk) spring shown in Fig 14 (b). How will you model the spring and what element would you choose? (6)

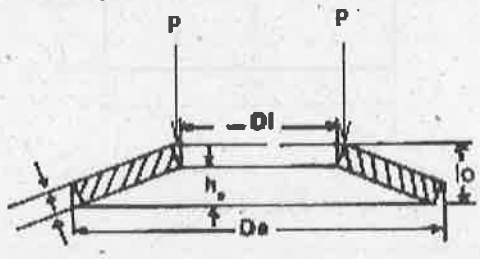

Fig. 14 (b)

15. (a) (i) Using Gauss Quadrature evaluate the following integral and compare with the exact value. (7)

$$I = \int_{-1}^{+1} \left(5\xi^3 - 4\xi^2 + 3\xi + 2\right) d\xi$$

(ii) Evaluate the shape functions for one corner node and one mid side node of a quadratic quadrilateral Serendipity element. (6)

Or

(b) (i) Why do we use natural coordinates? Differentiate between subparametric, isoparametric and superparametric elements. (5)

(ii) For the four noded element shown in Fig. 15(b) determine the Jacobian and evaluate its value at the point (1/3, 1/3). (8)

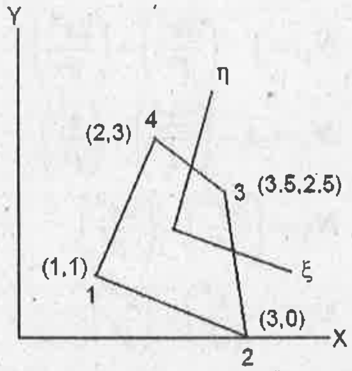

Fig. 15 (b)

PART C — (1 × 15 = 15 marks)

16. (a) A beam with dimensions shown in Fig. 16 (a) is attached to a machine. If resonance occurs what will be the first mode of vibration? Why is this mode occurring first? Determine the fundamental frequency of the beam. (15)

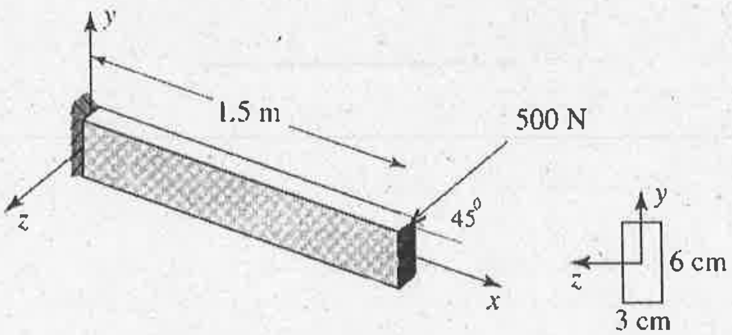

Fig. 16 (a)

Or

(b) A shaft of length 2000 mm and diameter 8 mm carries a pulley weighing 100 N at the center. If the shaft is mounted on both ends in self aligning ball bearings, determine the first natural frequency and plot the mode shape. $E = 200$ GPa and $\rho = 0.78 \times 10^6$ kg/m³.

$$\text{Stiffness Matrix } [K]^e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$[M]^e = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

$$\{f\}^e = \frac{ql}{2}\begin{Bmatrix} 1 \\ l/6 \\ 1 \\ -l/6 \end{Bmatrix}$$

$$N_1 = 1 - \left(\frac{3x^2}{l^2}\right) + \left(\frac{2x^3}{l^3}\right)$$

$$N_2 = x - \left(\frac{2x^2}{l}\right) + \left(\frac{x^3}{l^2}\right)$$

$$N_3 = \left(\frac{3x^2}{l^2}\right) - \left(\frac{2x^3}{l^3}\right)$$

$$N_4 = -\left(\frac{x^2}{l}\right) + \left(\frac{x^3}{l^2}\right)$$

No.of points	Location	Weight W_i
1	$\xi_1 = 0.00000$	2,00000
2	$\xi_1, \xi_2 \pm 0.57735$	1.00000
3	$\xi_1, \xi_3 = \pm 0.77459$	0.55555
	$\xi_2 = 0.00000$	0.88888